satisfying the balance of mass, momentum, and energy across any discontinuity. Similar results may be obtained by the procedure described here for the full Navier-Stokes equations for which the jump conditions are known; see, e.g., Green and Naghdi.[5]

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# Frequency Characteristics of Discrete Tones Generated in a High Subsonic Jet


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## Introduction

THE generation mechanism of jet noise was explained theoretically by Lighthill[1] using an idea of quadrupole oscillation of turbulent vortices in a jet; since then, many investigators have reported on sound waves that are emitted when a solid body is placed in a jet. Sometimes it is observed that strong sound waves with particular frequencies are emitted when a solid body is placed in a jet flow. These sound waves with narrow frequency band are called discrete tones. For the generation of such discrete tones, it is required that some proper mechanism exist for a flowfield to resonate at a particular frequency. Discrete tones such as edge tones,[2] blade tones,[3] and cavity tones[4] belong to the same category. Recently, the discrete tones radiated from the high subsonic jet impinging on a flat plate[5] have been investigated.

In our investigation, a new type of discrete tone is found experimentally. This discrete tone is radiated when a slender circular cylinder or a thin flat plate is placed in a high subsonic jet at right angles to the jet axis near the circular nozzle exit. The frequency characteristics of this discrete tone are different in some respects from those of other types of discrete tones investigated previously. This paper describes the frequency characteristics of this discrete tone found in our experiment.

## Experimental Apparatus

A schematic view of the experimental apparatus for the generation of the discrete tone is shown in Fig. 1a. A high subsonic air jet was exhausted from a circular nozzle with an internal diameter $d = 1.0$ cm. The jet-exhausted Mach number was regulated by a control valve. A slender circular cylinder or a thin flat plate was mounted on a traverse mechanism set along the jet axis ($\xi$ axis). The nozzle-to-cylinder (or plate) distance $x$ was remotely controlled by a pulse motor. The cross sections of the cylinder and the plate are shown in Fig. 1b. The system was located in a simplified anechoic chamber, where urethan foam was applied to the interior walls and the floor.

Measurements of sound pressure were made by using a 0.32-cm-diam Bruel and Kjaer type-4135 condenser microphone. The microphone was located at a fixed position in the backward arc about 50 deg from the jet axis, at a radius of 1.35 m from the nozzle exit, and 1.50 m above the floor. The experiments for the frequency characteristics of this discrete tone were carried out for the following conditions:

1) The Mach number $M$ of the jet at the nozzle exit is fixed, and the nozzle-to-cylinder (or plate) distance $x$ is changed.

2) The nozzle-to-cylinder (or plate) distance $x$ is fixed, and the flow Mach number $M$ of the jet is changed.

## Results and Discussion

Figure 2a shows a spectrum of the jet noise that has broad frequency band only. When the discrete tones are radiated, the spectrum has some sharp peaks at the resonance frequencies in addition to the broad frequency band as shown in Fig. 2b. The dominant frequencies of sound waves in the spectral curve are called the resonance frequencies and are denoted by $f$. In Fig. 3a, the resonance frequencies $f$ of the discrete tone were plotted against the nozzle-to-cylinder distance $x/d$ for $M = 0.886$, where $d$ is the nozzle diameter. In Fig. 3b, the resonance frequencies $f$ of the discrete tone were plotted against the flow Mach number $M$ for $x/d = 2.0$. It is observed that the results for the flat plate are very similar to the corresponding results for the cylinder. In these figures, black circles represent the frequencies of sounds with prominent peaks whose sound pressure levels exceed those of broad frequency band by 10 dB or more, and white circles represent the frequencies of sounds with small peaks whose sound pressure level exceed those of broad frequency band by 5-10 dB. From these experimental results shown in Figs. 3a and b, it is found that the discrete tone has the following characteristics:

1) This tone is generated when the nozzle-to-cylinder distance $x/d$ is in the range 0.5-8.0 and a jet is operated at $M \geq 0.6$.

2) When the Mach number $M$ of a jet at the nozzle exit is fixed, the observed resonance frequency of the discrete tones decreases gradually as the nozzle-to-cylinder distance is increased. The resonance frequency decreases until a certain

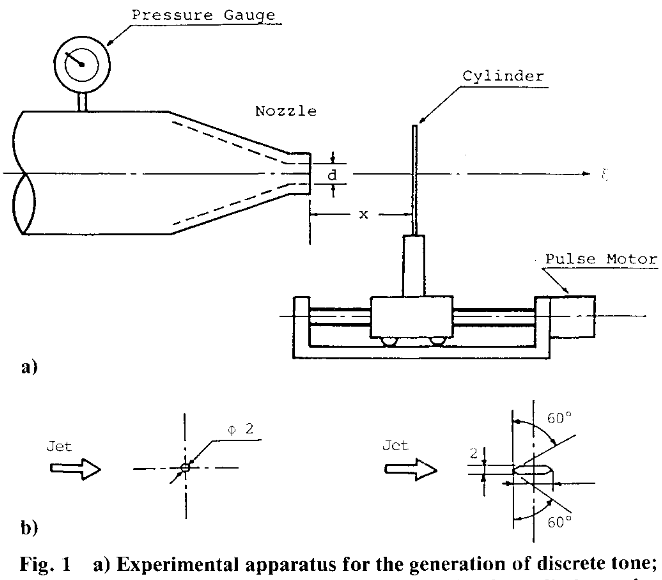


**Fig. 1 a) Experimental apparatus for the generation of discrete tone; ($\xi$: flow direction); b) geometry of a slender circular cylinder and a thin plate (arrow shows flow direction).**

Received June 17, 1985. 


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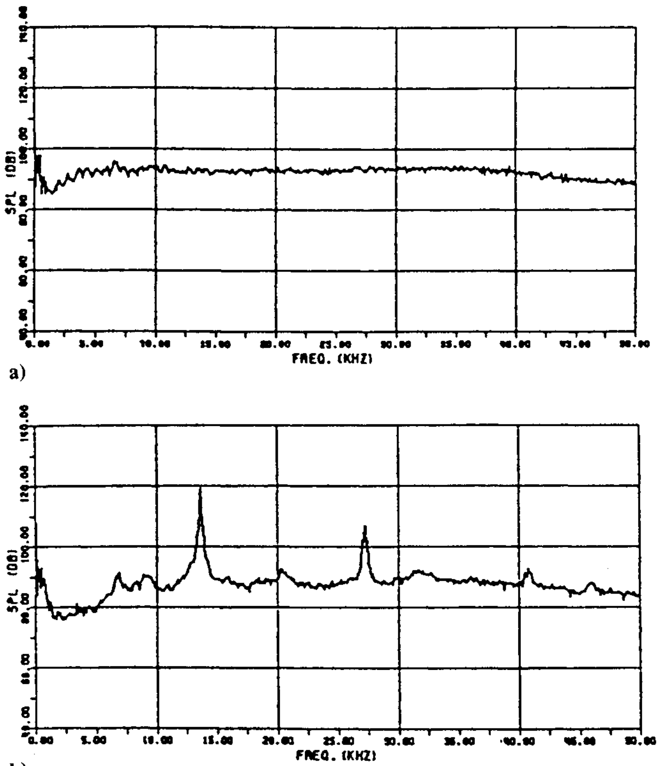


Fig. 2 Spectrum of sound wave: a) jet noise and b) discrete tone.

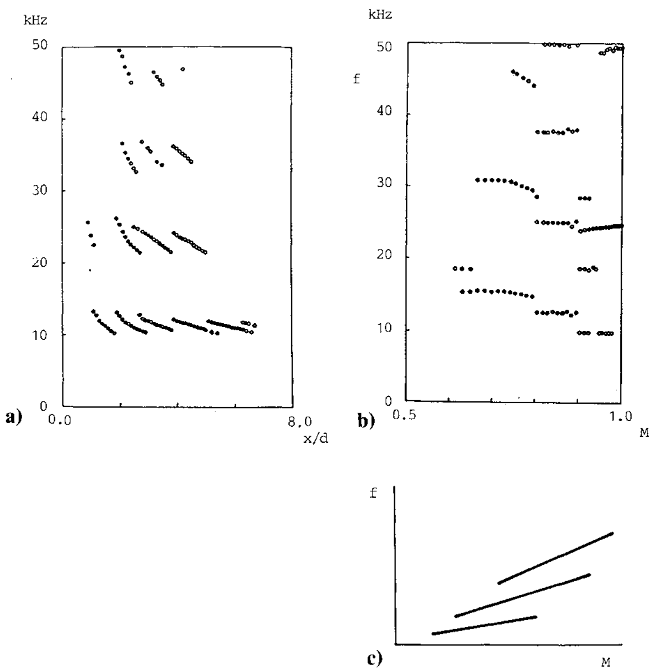


Fig. 3 a) Resonance frequency vs nozzle-to-cylinder distance for fixed flow Mach number $M = 0.886$; b) resonance frequency vs flow Mach number for fixed nozzle-to-cylinder distance $x/d = 2.0$ ($x = 2.0$ cm); c) frequency characteristics of edge tones at fixed slit-to-wedge distance obtained by Curle.

nozzle-to-cylinder distance is reached. A further increase in the distance results in a frequency jump to a higher mode of oscillation. Several modes of oscillation are demonstrated in Fig. 3a.

3) If the flow Mach number of the jet at the nozzle exit is increased gradually for a fixed nozzle-to-cylinder distance, the resonance frequency is kept nearly constant. As the Mach

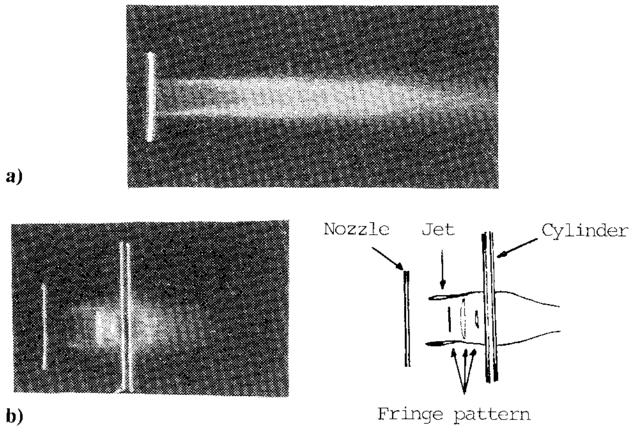


Fig. 4 Schlieren photographs of jet flow: a) jet only, $M = 0.886$; b) jet with cylinder, $x/d = 2.4$ ($x = 2.4$ cm), $M = 0.886$.

number is further increased, it jumps to a lower mode of oscillation at a certain value of Mach number. A few modes of oscillation are shown in Fig. 3b.

4) Several overtones are radiated in the frequency range below 50 kHz.

From the above observations, it can be said that the present frequency characteristics are somewhat different from those of other types of discrete tones previously investigated: e.g., when the nozzle-to-cylinder (or plate) distance is changed at a fixed flow Mach number, the frequency characteristics of the present discrete tone are similar to those of the edge tones and of the discrete tones radiated from the high subsonic jet impinging on a flat plate. But, when the flow Mach number is changed and the position of a cylinder or a plate is fixed, the frequency characteristics of this discrete tone are different from those of the edge tones as shown in Fig. 3c, where the frequency increases and jumps to a higher frequency as the flow Mach number gradually increases, and are also different from other types of discrete tones investigated previously.

Figure 4 shows schlieren photographs of jet flows. Figure 4a is a photograph of the jet before a cylinder or a flat plate is placed in it for the flow Mach number $M = 0.886$. Figure 4b is a photograph of the jet flow where the cylinder is placed at the distance $x/d = 2.4$ ($x = 2.4$ cm) and the flow Mach number $M$ is 0.886. In the latter case, the discrete tone is radiated. In the present visualization, various kinds of fringe patterns were observed in the jet, depending on the flow Mach number $M$ and the nozzle-to-cylinder distance $x/d$. These fringe patterns are almost perpendicular to the jet axis.

## Conclusions

In this report, it is shown that a new type of discrete tone is radiated when a cylinder or a thin flat plate is placed in a high subsonic jet at right angles to the jet axis in the vicinity of a circular nozzle. The frequency characteristics of this discrete tone are different in some respects from those of other types of discrete tones such as edge tones. The fringe patterns in the schlieren photographs are found in the jet where the discrete tones are radiated. The authors are now investigating the relation between the frequency of this discrete tone and the fringe patterns in the jet in order to clarify the generation mechanism of this discrete tone. The result will be reported in the near future.

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# Mach Reflection and Aerodynamic Choking in Two-Dimensional Ducted Flow

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## Introduction

IT is well known in the quasi-one-dimensional (1-D) theory for flow through a converging-diverging duct that, for a given throat to inflow area ratio $A_T/A_i$, there exists a supersonic inflow Mach number $M_i$ below which the duct will choke aerodynamically (see, for example, Ref. 1). This Mach number $M_i$, within the assumptions of quasi-1-D flow, does not depend on how the change in inflow area to throat area is distributed in the axial direction. Therefore, it is possible to construct a single curve of $A_T/A_i$ as a function of the smallest value of supersonic inflow Mach number $M_i$, which provides sonic throat conditions for quasi-1-D flow. This curve can then be used to determine whether a duct with a certain area ratio will choke or not for a given inflow Mach number. (This Note considers only the supersonic inflow conditions although a similar curve can be constructed for the subsonic inflow conditions as well.) In the present study, flow through a two-dimensional (2-D) duct with supersonic inflow is investigated numerically from the point of view of formation of Mach reflection, aerodynamic choking, and the possibility of constructing a curve similar to that for the quasi-1-D flow discussed above.

## Results and Discussion

The Mach numbers and area ratios for which the calculations are made are tabulated in Table 1. The area ratio is varied by moving the lower boundary, AB, up or down. A grid of $55 \times 61$ points is used in the calculations. Results of these calculations are shown in Figs. 3-5.

Figure 3 shows the pressure contours in the first configuration at various inflow Mach numbers and at an area ratio of 0.7085. It is seen from this figure that at Mach 2.2 the shock wave undergoes regular or simple reflections, but as the inflow Mach number is decreased to 2.1 formation of a Mach reflection takes place at the lower boundary. This kind of reflection occurs when the Mach number behind the incident shock is lower than the detachment Mach number for the angle of flow turning on the incident boundary. Under such a situation, no simple shock reflection is possible thus causing a portion of the flow to pass through a normal or nearly normal shock that appears near the incident boundary. It also produces a small region of subsonic flow behind the normal shock. As the Mach number is decreased further to 2.0 and 1.95, the extent of Mach reflection is seen to grow in size thus producing a larger and larger region of subsonic flow. At Mach 1.9, the duct flow chokes causing a normal shock to stand slightly upstream of the duct entrance. Under the choked conditions, a portion of the flow approaching the duct is spilled around the leading edge of the duct upper boundary and the flow in the duct is again established with subsonic inflow and sonic throat.

Figure 4 shows the mass flux contours at Mach 1.95 and 1.9. The spilling of the flow at Mach 1.9 can be seen clearly from this figure. Also notice from Figs. 3 and 4 that no interaction between the flow through the duct and the flow in the extended region takes place until the inflow Mach number is reduced so as to choke the duct. Similar calculations were also made at area ratios of 0.638 and 0.7989 for the first configuration. Based on these calculations, Fig. 5 shows line plots of the inflow Mach numbers at which the duct chokes and the duct flow starts undergoing simple shock reflections against the area ratio. Also shown in this figure is a plot of the inflow Mach number at which the quasi-1-D flow chokes. It is seen that the 2-D flow chokes at a higher Mach number than the quasi-1-D flow for a given area ratio. Further, there is a distinct Mach number range above the choking Mach number in which the 2-D flow undergoes a Mach reflection. For example, at the area ratio of 0.7085, the quasi-1-D flow chokes at Mach 1.775 and the 2-D flow chokes around Mach 1.9. Between Mach 1.9 and 2.2, the 2-D flow undergoes a Mach reflection. In contrast, calculations for the second configuration at the same area ratio showed that choking occurred around Mach 1.7925 and simple shock reflections started taking place at Mach 1.825. Based on the preceding calculations for the two 2-D ducts, the following conclusions can be drawn.

## Analysis

In the present investigation, two 2-D configurations, shown in Fig. 1, are analyzed. The first configuration has a large in-

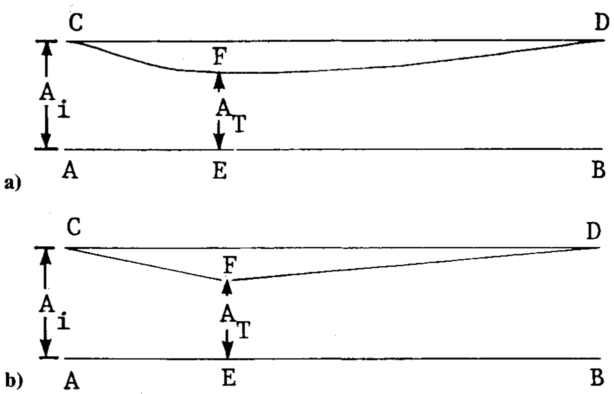


**Fig. 1 a) Geometry of the first and b) second configurations.**

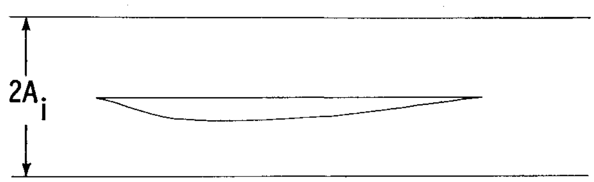


**Fig. 2 Physical domain of computation.**

**Table 1 Area ratios and inflow Mach numbers**

| $A_T/A_i$ | Inflow Mach numbers for First configuration | Inflow Mach numbers for Second configuration |
|---|---|---|
| 0.638 | 2.05, 2.1, 2.15, 2.25, 2.3 | — |
| 0.7085 | 1.9, 1.95, 2.0, 2.1, 2.2 | 1.7925, 1.8, 1.825 |
| 0.7985 | 1.75, 1.8, 1.9, 2.0, 2.05 | — |

Received July 18, 1985; revision received Aug. 19, 1985. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

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